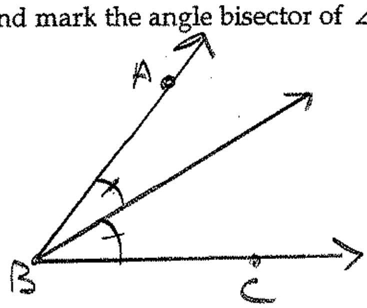
Week 5:

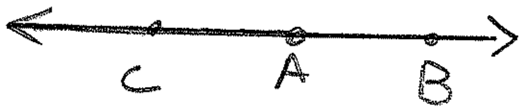
Parallel and Perpendicular Lines
Quadrilaterals: Parallelograms, Trapezoids and Kites
Area of Quadrilaterals and Triangles
Polygons

1. Draw 3 collinear points, ***A, B, C***.

2. Draw and mark the angle bisector of $\angle ABC$.

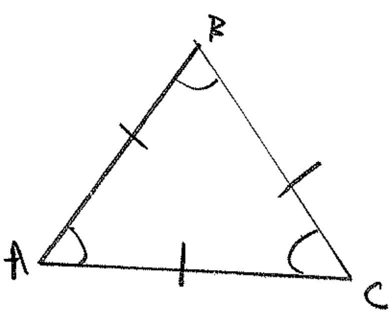


3. Draw and identify one pair of opposite rays.



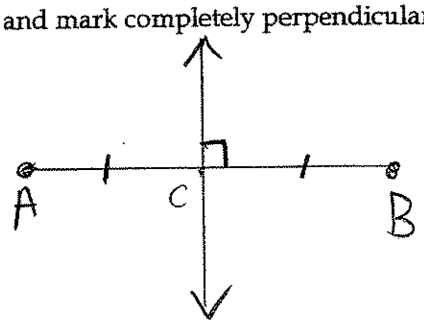
$\overrightarrow{AB}$, $\overrightarrow{AC}$ } are opposite rays

4. Draw and mark completely equilateral triangle *ABC*.



equiangular

5. Draw and mark completely perpendicular bisector on segment ***AB***.



$AC = CB$

$\overline{AC} \cong \overline{CB}$

Name: ______________________________ Date: __________

Geometry: Lines and Angles

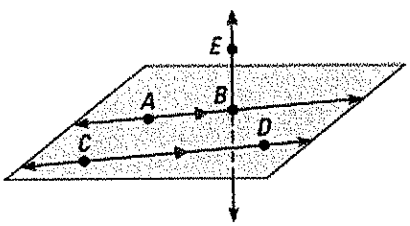
I. Relationships between lines

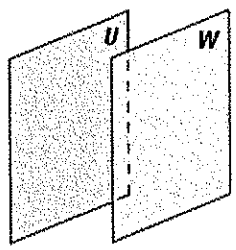
<u>Vocabulary</u>

Parallel lines: Two coplanar lines that **do not** intersect.

Skew lines: lines that are **not** coplanar and **do not** intersect.

Parallel Planes: Two planes that **do not** intersect.

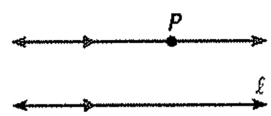




PARALLEL AND PERPENDICULAR POSTULATES

POSTULATE 13 *Parallel Postulate*

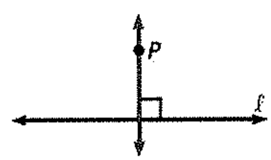
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through *P* parallel to ℓ.

POSTULATE 14 *Perpendicular Postulate*

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

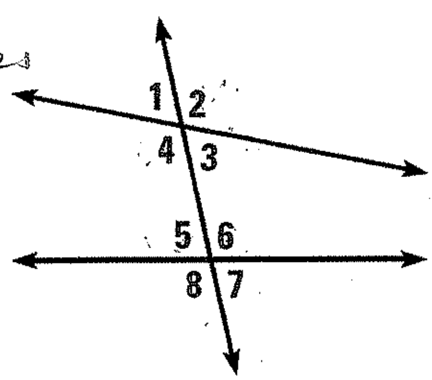


There is exactly one line through *P* perpendicular to ℓ.

II. Identifying Angles Formed by Transversals

<u>Vocabulary</u>

Transversal: a line that intersect 2 other lines



Corresponding angles:

∠2+∠6, ∠1+∠5, ∠4+∠8, ∠3+∠7

Alternate Interior Angles:

∠4+∠6, ∠3+∠5

Alternate Exterior Angles:

∠2+∠8, ∠1+∠7

Consecutive Interior Angles (Same Side Interior Angles):

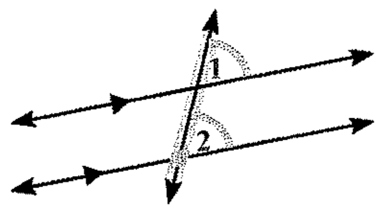
∠3+∠6, ∠4+∠5

GOAL 1 PROPERTIES OF PARALLEL LINES

POSTULATE

POSTULATE 15 Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



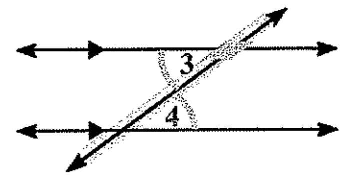
$\angle 1 \cong \angle 2$

GOAL 1 PROPERTIES OF PARALLEL LINES

THEOREMS ABOUT PARALLEL LINES

THEOREM 3.4 Alternate Interior Angles

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.



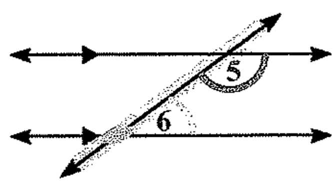
$\angle 3 \cong \angle 4$

GOAL 1 PROPERTIES OF PARALLEL LINES

THEOREMS ABOUT PARALLEL LINES

THEOREM 3.5 Consecutive Interior Angles

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



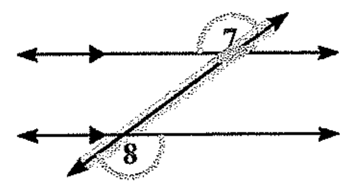
$m\angle 5 + m\angle 6 = 180°$

GOAL 1 PROPERTIES OF PARALLEL LINES

THEOREMS ABOUT PARALLEL LINES

THEOREM 3.6 Alternate Exterior Angles

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



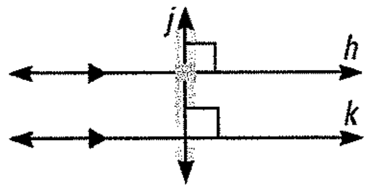
$\angle 7 \cong \angle 8$

GOAL 1 PROPERTIES OF PARALLEL LINES

THEOREMS ABOUT PARALLEL LINES

THEOREM 3.7 Perpendicular Transversal

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.



$j \perp k$

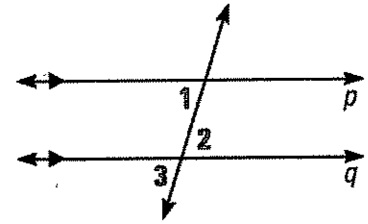
EXAMPLE Proving the Alternate Interior Angles Theorem

Prove the Alternate Interior Angles Theorem.

SOLUTION

GIVEN $p \parallel q$

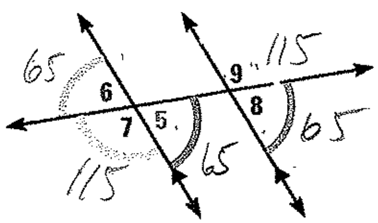
PROVE $\angle 1 \cong \angle 2$



Statements	Reasons
1 $p \parallel q$	1 Given
2 $\angle 1 + \angle 3$ are corr. ∡	2 Def of corr. ∡
3 $\angle 1 \cong \angle 3$	3 Corr. ∡ Postulate
4 $\angle 2 + \angle 3$ are vertical ∡	4 Def of vertical ∡
5 $\angle 2 \cong \angle 3$	5 Vertical Angle Theorem
6 $\angle 1 \cong \angle 2$	6 Substitution

EXAMPLE Using Properties of Parallel Lines

Given that $m\angle 5 = 65°$, find each measure. Tell which postulate or theorem you use.



SOLUTION

$\angle 7 + \angle 9$ are alt. ext. ∡

$\angle 5 + \angle 8$ are corr. ∡

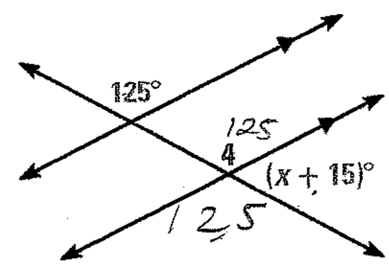
$\angle 5 + \angle 6$ are vert. ∡

$\angle 6 + \angle 7$ are supp.

GOAL 2 PROPERTIES OF SPECIAL PAIRS OF ANGLES

EXAMPLE Using Properties of Parallel Lines

Use properties of parallel lines to find the value of x.



SOLUTION

$125 + x + 15 = 180$

$x + 140 = 180$

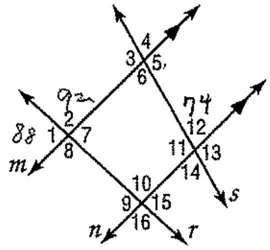
$x = 40°$

NAME ______________________ DATE __________ PERIOD _____

3-2 Practice

Angles and Parallel Lines

In the figure, $m\angle 2 = 92$ and $m\angle 12 = 74$. Find the measure of each angle.



1. $\angle 10$ 92°

2. $\angle 8$ 92°

3. $\angle 9$ 88°

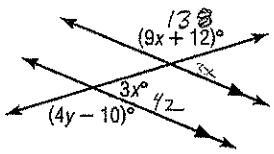
4. $\angle 5$ 106°

5. $\angle 11$ 106°

6. $\angle 13$ 106°

Find x and y in each figure.

7.



4y−10 = 138

4y = 148

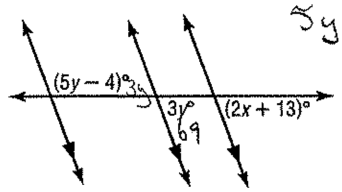
y = 37°

9x+12+3x = 180

12x = 168

x = 14

8.



5y−4+3y = 180

8y−4 = 180

8y = 184

y = 23

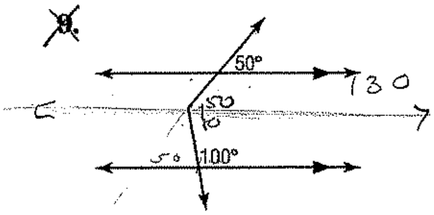
2x+13 = 69

2x = 56

x = 28

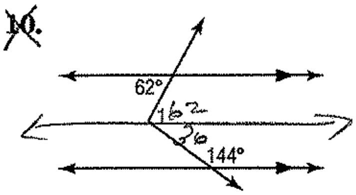
Find $m\angle 1$ in each figure.

~~**9.**~~



130

~~**10.**~~

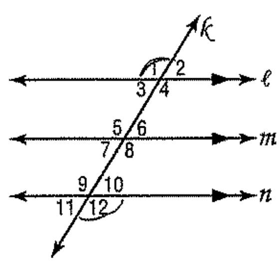


98

11. PROOF Write a ~~paragraph~~ 2 column proof of ~~Theorem 3.3.~~

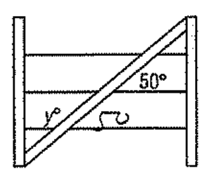
Given: $\ell \parallel m$, $m \parallel n$

Prove: $\angle 1 \cong \angle 12$



① $\ell \parallel m$	① given
② $\angle 1 \cong \angle 5$	② Corr ∡ Post.
③ $m \parallel n$	③ given
④ $\angle 5 \cong \angle 12$	④ Alt. Ext. ∡ Th.
⑤ $\angle 1 \cong \angle 12$	⑤ Transitive

12. FENCING A diagonal brace strengthens the wire fence and prevents it from sagging. The brace makes a 50° angle with the wire as shown. Find y.



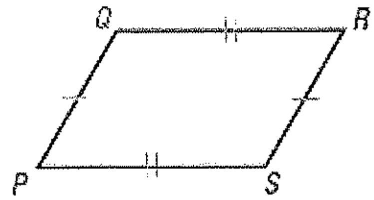
y = 130

THEOREMS ABOUT PARALLELOGRAMS

THEOREM 6.2

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

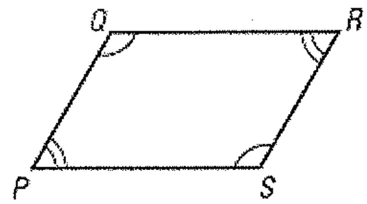
$\overline{PQ} \cong \overline{RS}$ and $\overline{SP} \cong \overline{QR}$



THEOREM 6.3

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

$\angle P \cong \angle R$ and $\angle Q \cong \angle S$

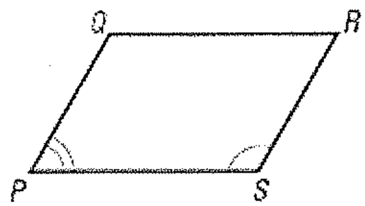


THEOREM 6.4

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

$m\angle P + m\angle Q = 180°$, $m\angle Q + m\angle R = 180°$,

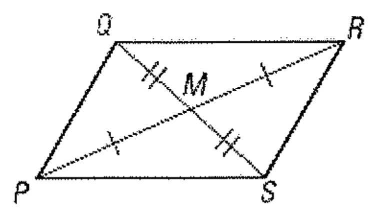
$m\angle R + m\angle S = 180°$, $m\angle S + m\angle P = 180°$



THEOREM 6.5

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$



PROVING QUADRILATERALS ARE PARALLELOGRAMS

- Show that both pairs of opposite sides are parallel.
- Show that both pairs of opposite sides are congruent.
- Show that both pairs of opposite angles are congruent.
- Show that one angle is supplementary to both consecutive angles.
- Show that the diagonals bisect each other.
- Show that one pair of opposite sides are congruent and parallel.

LESSON **6.2**

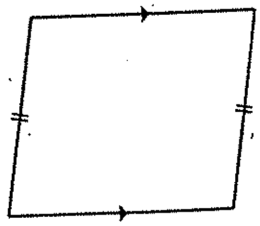
NAME ______________________________ DATE __________

Practice B

For use with pages 330–337

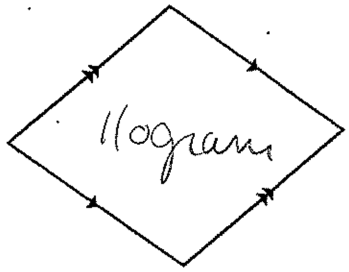
Decide whether the figure is a parallelogram. If it is not, explain why not.

1.

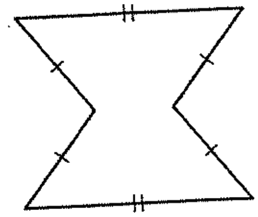


Isosceles trap.

2.

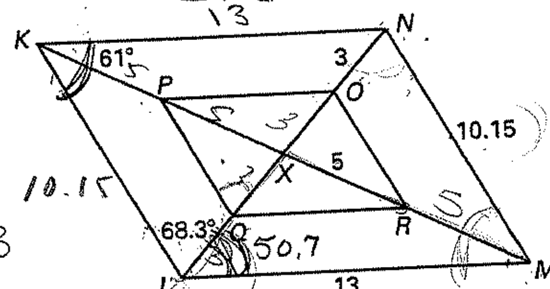


3.



No, more than 4 sides

Use the diagram of parallelogram *KLMN* at the right. Points *O, P, Q, R* are midpoints of $\overline{XN}$, $\overline{XK}$, $\overline{XL}$, and $\overline{XM}$. Find the indicated measures.



4. *KN* 13

5. *KL* 10.15

6. *XN* 6

7. *LN* 12

8. *KP* 5

9. *KR* 15

10. $m\angle MNL$ 68.3

11. $m\angle NLM$ 50.7

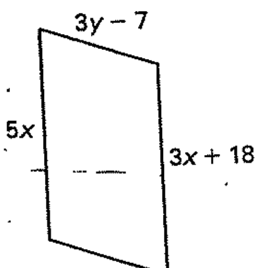
12. $m\angle NML$ 61

13. $m\angle XQP$ 68.3

14. Perimeter of parallelogram *KLMN* 46.3

Find the value of each variable in the parallelogram.

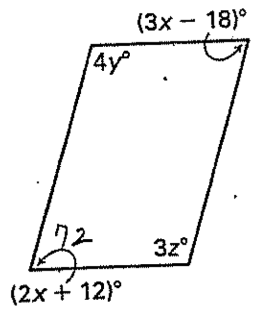
15.



$5x = 3x + 18$
$2x = 18$
$x = 9$

$3y - 7 = 2y + 4$
$y - 7 = 4$
$y = 11$

16.

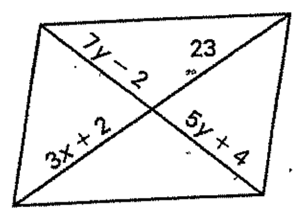


$2x + 12 = 3x - 18$
$2x + 30 = 3x$
$30 = x$

$4y + 72 = 180$
$4y = 108$
$y = 27$

$3z + 72 = 180$
$3z = 108$
$z = 36$

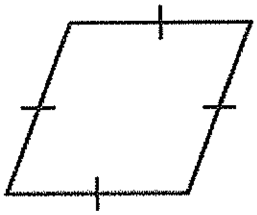
17.



$3x + 2 = 23$
$3x = 21$
$x = 7$

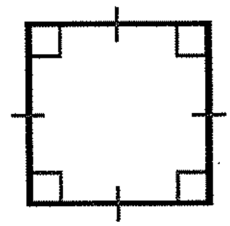
$7y - 2 = 5y + 4$
$2y - 2 = 4$
$2y = 6$
$y = 3$

Special Parallelograms!

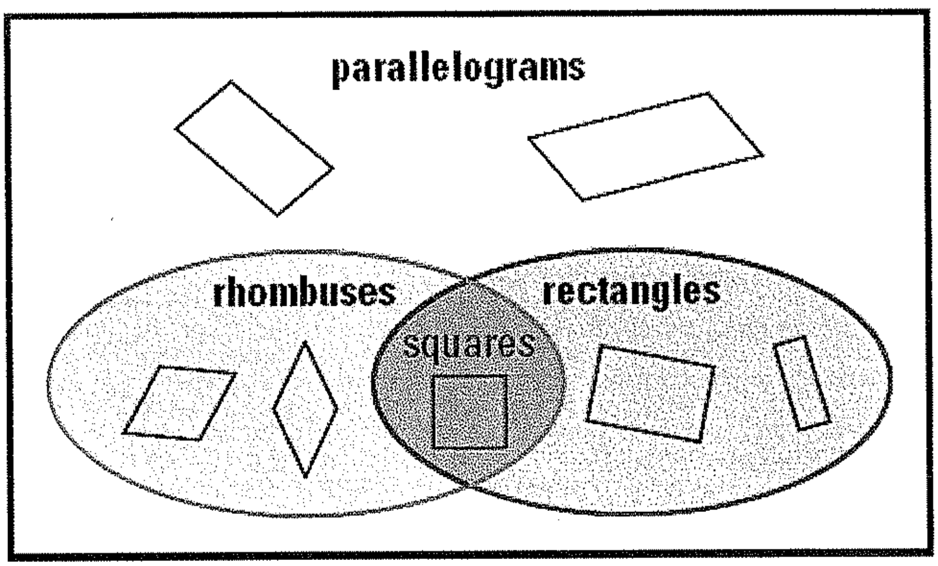


A **rhombus** is a parallelogram with four congruent sides.

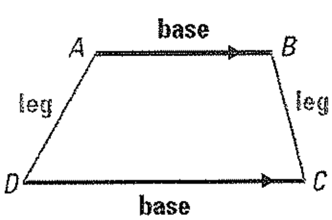
A **rectangle** is a parallelogram with four right angles.



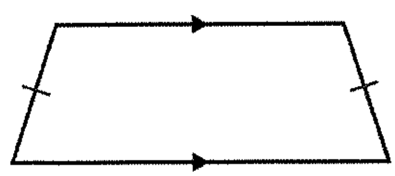
A **square** is a parallelogram with four congruent sides and four right angles.



Trapezoids and Kites!

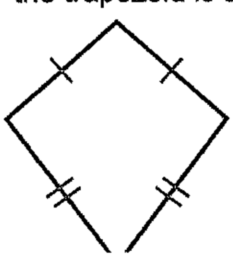


A trapezoid is a quadrilateral with exactly one pair of parallel sides.



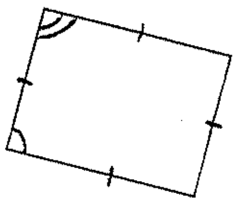
isosceles trapezoid

If the legs of a trapezoid are congruent, then the trapezoid is an isosceles trapezoid.



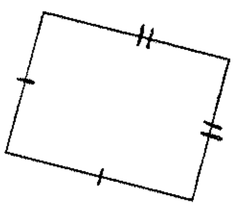
A kite is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are **not** congruent.

1. Identify the quadrilateral.



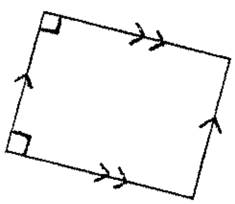
1. rhombus

2. Identify the quadrilateral.



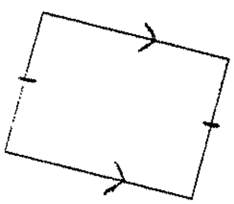
2. Kite

3. Identify the quadrilateral.



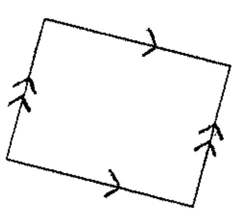
3. rectangle

4. Identify the quadrilateral.



4. isosceles trapezoid

5. Identify the quadrilateral.



5. //ogram

Properties of Quadrilaterals

(Put a check in each box where the property is always true)

	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles Trapezoid	Kite
Both pairs of opposite sides are parallel.	X	X	X	X			
Both pairs of opposite sides are congruent.	X	X	X	X			
Exactly one pair of opposite sides is parallel					X	X	
Exactly one pair of opposite sides is congruent.						X	
The diagonals bisect each other.	X	X	X	X			
The diagonals are congruent.		X		X		X	
The diagonals are perpendicular.			X	X			X
Both pairs of opposite angles are congruent	X	X	X	X			
Exactly one pair of opposite angles is congruent							X
Consecutive angles are supplementary.	X	X	X	X			
All sides are congruent.			X	X			
All angles are congruent		X		X			

6.7 NOTES: AREA OF TRIANGLES AND QUADRILATERALS

AREA POSTULATES

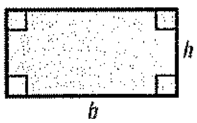
Postulate 22 Area of a Square Postulate
The area of a square is the square of the length of its side, or $A = s^2$

Postulate 23 Area Congruence Postulate
If two polygons are congruent, then they have the same area.

Postulate 24 Area Addition Postulate
The area of a region is the sum of the areas of its nonoverlapping parts.

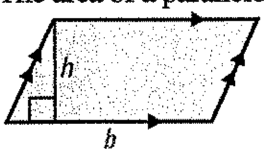
AREA THEOREMS

Theorem 6.20 Area of a Rectangle
The area of a rectangle is the product of its base and height.



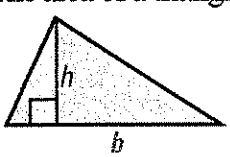
$A = bh$

Theorem 6.21 Area of a Parallelogram
The area of a parallelogram is the product of a base and its corresponding height.



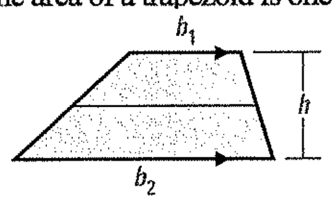
$A = bh$

Theorem 6.22 Area of a Triangle
The area of a triangle is one half the product of a base and its corresponding height.



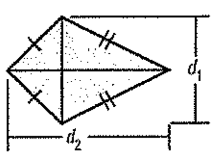
$A = \frac{1}{2}bh$

Theorem 6.23 Area of a Trapezoid
The area of a trapezoid is one half the product of the height and the sum of the bases.



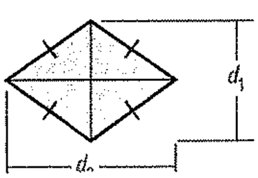
$A = \frac{1}{2}(b_1 + b_2)h$

Theorem 6.24 Area of a Kite
The area of a kite is one half the product of the lengths of its diagonals.



$A = \frac{1}{2}d_1 d_2$

Theorem 6.25 Area of a Rhombus
The area of a rhombus is equal to one half the product of the lengths of the diagonals.



$A = \frac{1}{2}d_1 d_2$

LESSON
6.7

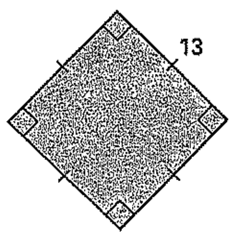
NAME ______________________ DATE __________

Practice B

For use with pages 372–380

Find the area of the polygon.

1.

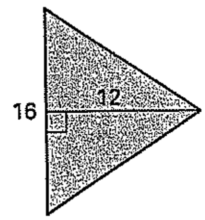


$A = s^2$

$= 13^2$

$= 169\ u^2$

2.

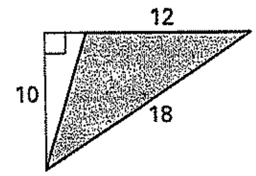


$A = \frac{1}{2}bh$

$= \frac{1}{2}(16)(12)$

$= 96\ u^2$

3.

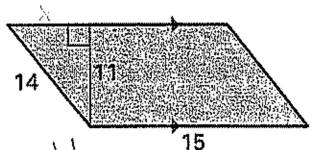


$A = \frac{1}{2}bh$

$= \frac{1}{2}(12)(10)$

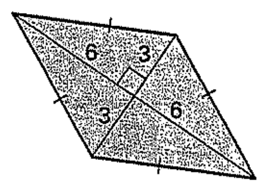
$= 60\ u^2$

4.



What's wrong here? don't know what it is!

5.

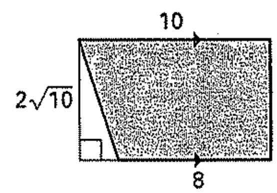


$A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(6)(12)$

$= 36\ u^2$

6.



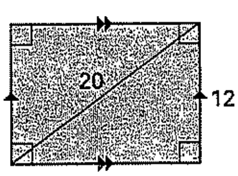
$A = \frac{1}{2}(b_1+b_2)h$

$= \frac{1}{2}(8+10)(2\sqrt{10})$

$= 9(2\sqrt{10})$

$18\sqrt{10} \approx 56.92\ u^2$

7.



$12^2 + x^2 = 20^2$

$144 + x^2 = 400$

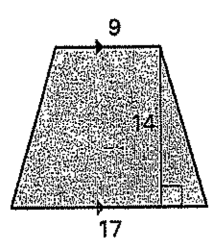
$x^2 = 256$

$x = 16$

$A = bh$

$= 16(12) = 96\ u^2$

8.



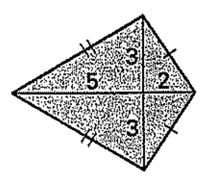
$A = \frac{1}{2}(b_1+b_2)h$

$= \frac{1}{2}(9+17)14$

$= 7(26)$

$= 182\ u^2$

9.



$A = \frac{1}{2}d_1d_2$

$= \frac{1}{2}(6)(7)$

$= 21\ u^2$

Lesson 6.7

Geometry

11.1 Angle Measurements in Polygons

If n is the number of sides in a polygon, then

- The sum of the interior angles is: (n-2)180
- The sum of the exterior angles is: 360

If the polygon is a **regular** polygon, then

- The measure of one interior angle is: (n-2)180/n
- The measure of one exterior angle is: 360/n

State the number of sides and the number of interior angles of the polygon.

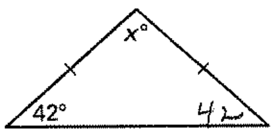
1. quadrilateral 4 **2.** hexagon 6 **3.** decagon 10 **4.** pentagon 5

Find the sum of the measures of the interior angles of the convex polygon.

5. hexagon 720 **6.** octagon 1080 **7.** 12-gon 1440 **8.** 15-gon 2340

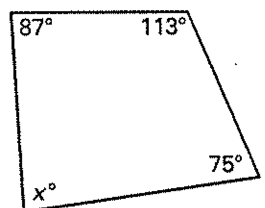
Find the value of *x*.

9.



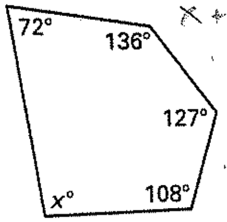
x + 42 + 42 = 180
x + 84 = 180
x = 96

10.



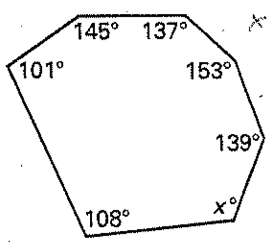
x + 87 + 113 + 75 = 360
x + 275 = 360
x = 85

11.



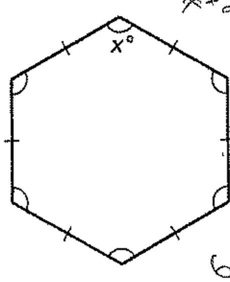
x + 72 + 136 + 127 + 108 = 540
x + 443 = 540
x = 97

12.



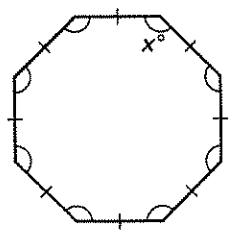
x + 783 = 900
x = 117

13.



6x = 720
x = 120

14.



8x = 1080
x = 135

You are given the measure of each interior angle of a regular *n*-gon. Find the value of *n*.

15. 90° (n-2)180/n = 90; 180n - 360 = 90n; 90n = 360; n = 4

16. 108° (n-2)180/n = 108; 108n = 180n - 360; 72n = 360; n = 5

17. 135° (n-2)180/n = 135; 180n - 360 = 135n; 360 = 45n; n = 8

18. 144° (n-2)180/n = 144; 180n - 360 = 144n; 360 = 36n; n = 10

Find the sum of the measures of the exterior angles of the convex polygon.

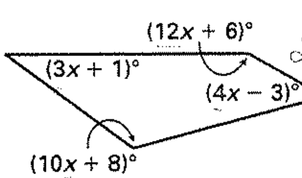
19. hexagon 360 **20.** octagon 360 **21.** 12-gon 360 **22.** 15-gon 360

You are given the measure of each exterior angle of a regular *n*-gon. Find the value of *n*.

23. 90° 90 = 360/n; n = 4 **24.** 60° 6 **25.** 45° 8 **26.** 30° 12

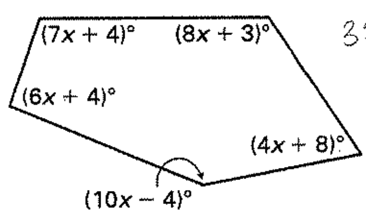
Find the value of *x*.

27.



29x + 12 = 360
29x = 348
x = 12

28.



35x + 15 = 540
35x = 525
x = 15

Lesson 11.1